The Invention is Claimed to Be:

1.   A method for enabling optimizing software to determine an optimal allocation in a combinatorial auction, the method comprising:

    (a)   receiving a plurality of bids each of which includes a plurality of sub bids, wherein each sub bid is comprised of one of (1) one good and an associated price and (2) a logical operator logically connecting at least two child sub bids and a price associated with the logical operator;

    (b)   defining an objective for the plurality of bids;

    (c)   defining for each bid a plurality of mathematical relationships without logical operators, wherein said mathematical relationships collectively represent the bid; and

    (d)   causing the optimizing software to process the received bids to achieve the objective subject to the mathematical relationships.

2.   The method of claim 1, wherein step (c) includes, for each sub bid comprised of one good and an associated price, defining:

    a first mathematical relationship between a pair of Boolean variables that relate (1) the one good being allocated to the bid that includes the sub bid to (2) satisfaction of the sub bid, wherein the sub bid is satisfied when the one good is allocated thereto; and

    a second mathematical relationship that relates (1) a value of the sub bid to (2) a product of the price of the sub bid times a value of a Boolean variable related to the satisfaction of the sub bid.

3.   The method of claim 2, wherein:

    the first mathematical relationship includes setting (1) the Boolean variable related to satisfaction of the sub bid less than or equal to ($\leq$) (2) the Boolean variable related to the bid including the sub bid being allocated the one good; and

the second mathematical relationship includes setting (1) the value of the sub bid ≤(2) the product of the price of the sub bid times the value of a Boolean variable related to the satisfaction of the sub bid.

4.   The method of claim 1, wherein step (c) includes, for each sub bid comprised of a logical operator AND logically connecting at least two child sub bids, defining:

a third mathematical relationship that relates (1) a sum of Boolean values related to satisfaction of each child sub bid to (2) a product of the total number of the child sub bids logically connected by the logical operator AND times a Boolean value related to the satisfaction of the sub bid comprised of the logical operator AND, wherein the sub bid comprised of the logical operator AND is satisfied when all of the child sub bids logically connected thereby are satisfied; and

a fourth mathematical relationship that relates (1) a value of the sub bid comprised of the logical operator AND to (2) a sum of the values of each child sub bid that is satisfied and the price associated with the sub bid comprised of the logical operator AND, wherein said price is included in the sum when said sub bid is satisfied, otherwise it is not included in the sum.

5.   The method of claim 4, wherein:

the third mathematical relationship includes setting (1) the product of the total number of the child sub bids logically connected by the logical operator AND times a Boolean value related to the satisfaction of the sub bid comprised of the logical operator AND ≤(2) the sum of the Boolean values related to satisfaction of each of the at least two child sub bids; and

the fourth mathematical relationship includes setting (1) the value of the sub bid comprised of the logical operator AND ≤(2) the sum of (i) the values of the at least two child sub bids and (ii) the price associated with the sub bid comprised of the logical operator AND times the Boolean value related to satisfaction of said sub bid.

6.   The method of claim 1, wherein step (c) includes, for each sub bid comprised of a logical operator OR or XOR logically connecting at least two child sub bids, defining:

a fifth mathematical relationship that relates (1) a sum of Boolean values related to satisfaction of each child sub bid to (2) satisfaction of the sub bid comprised of the logical operator OR or XOR, wherein the sub bid comprised of the logical operator OR or XOR is satisfied when at least one of the child sub bids logically connected thereby is satisfied, and

a sixth mathematical relationship that relates (1) a value of the sub bid comprised of the logical operator OR or XOR to (2) a sum of the values of each child sub bid that is satisfied and the price associated with the sub bid comprised of the logical operator OR or XOR, wherein said price is included in the sum when said sub bid is satisfied, otherwise it is not included in the sum.

7.   The method of claim 6, wherein:

the fifth mathematical relationship includes setting (1) the satisfaction of the sub bid comprised of the logical operator OR or XOR $\leq$(2) the sum of Boolean values related to satisfaction of each of the at least two child sub bids; and

the sixth mathematical relationship includes setting (1) the value of the sub bid comprised of the logical operator OR or XOR $\leq$(2) the sum of the values of the at least two child sub bids and the price associated with the sub bid comprised of the logical operator OR or XOR times the Boolean value related to satisfaction of said sub bid.

8.   The method of claim 1, wherein step (c) includes, for each sub bid comprised of a logical operator XOR logically connecting the at least two child sub bids, defining a seventh mathematical relationship that relates (1) an integer value to (2) a sum of Boolean values related to each child sub bid, wherein each child sub bid that contributes value to the sub bid comprised of the logical operator XOR is assigned a first Boolean value, otherwise it is assigned a second Boolean value.

9.   The method of claim 8, wherein the seventh mathematical relationship includes setting (1) the sum of the Boolean values related to the at least two child sub bids $\leq$(2) the integer value.

10.   The method of claim 1, wherein step (c) includes defining an eighth mathematical relationship for each child sub bid that contributes value to the sub bid comprised of the logical operator XOR, wherein said relationship relates (1) a value of the child sub bid to (2) a product of the Boolean value of said child sub bid times a predetermined value.

11.   The method of claim 10, wherein the eighth mathematical relationship includes setting (1) the value of the child sub bid $\leq$ (2) the product of the Boolean value of said sub bid times the predetermined value.

12.   The method of claim 10, wherein the predetermined value is greater than or equal to the largest value of any of the child sub bids that contributes value to the sub bid comprised of the logical operator XOR.

13.   The method of claim 12, wherein the predetermined value is the sum of all the prices included in the bid including the child sub bids.

14.   The method of claim 1, wherein step (c) includes, for each sub bid for k number of child sub bids, where k is less than a total number of child sub bids available, defining:

a ninth mathematical relationship that relates (1) a total number of satisfied child sub bids bid to (2) a sum of Boolean values related to satisfaction of each child sub bid;

a tenth mathematical relationship that relates (1) a total number of satisfied child sub bids to (2) a product of k times a Boolean value related to satisfaction of the sub bid; and

an eleventh mathematical relationship that relates (1) a value of the sub bid to (2) a sum of the values of each child sub bid that is satisfied and a price associated with the sub

bid, wherein said price is included in the sum when said sub bid is satisfied, otherwise it is not included in the sum.

15.   The method of claim 15, wherein

the ninth mathematical relationship includes setting (1) the total number of satisfied child sub bids $\leq$(2) the sum of Boolean values related to satisfaction of each child sub bid;

the tenth mathematical relationship includes setting (1) the product of k times the Boolean value related to satisfaction of the sub bid $\leq$(2) the total number of satisfied child sub bids; and

the eleventh mathematical relationship includes setting (1) the value of the sub bid $\leq$(2) the sum of the values of each child sub bid that is satisfied and the price associated with the sub bid times a Boolean value related to satisfaction of the sub bid.

16.   The method of claim 1, wherein step (c) includes:

for each sub bid comprised of one good and an associated price, defining:

a first mathematical relationship between a pair of Boolean variables that relate (1) the one good being allocated to the bid that includes the sub bid to (2) satisfaction of the sub bid, wherein the sub bid is satisfied when the one good is allocated thereto, and

a second mathematical relationship that relates (1) a value of the sub bid to (2) a product of the price of the sub bid times a value of a Boolean variable related to the satisfaction of the sub bid;

for each sub bid comprised of a logical operator AND logically connecting at least two child sub bids, defining:

a third mathematical relationship that relates (1) a sum of Boolean values related to satisfaction of each child sub bid to (2) a product of the total number of the child sub bids logically connected by the logical operator AND times a Boolean value related to the satisfaction of the sub bid comprised of the logical operator AND, wherein the sub bid comprised

of the logical operator AND is satisfied when all of the child sub bids logically connected thereby are satisfied, and

a fourth mathematical relationship that relates (1) a value of the sub bid comprised of the logical operator AND to (2) a sum of the values of each child sub bid that is satisfied and the price associated with the sub bid comprised of the logical operator AND, wherein said price is included in the sum when said sub bid is satisfied, otherwise it is not included in the sum;

for each sub bid comprised of a logical operator OR or XOR logically connecting at least two child sub bids, defining:

a fifth mathematical relationship that relates (1) a sum of Boolean values related to satisfaction of each child sub bid to (2) satisfaction of the sub bid comprised of the logical operator OR or XOR, wherein the sub bid comprised of the logical operator OR or XOR is satisfied when at least one of the child sub bids logically connected thereby is satisfied, and

a sixth mathematical relationship that relates (1) a value of the sub bid comprised of the logical operator OR or XOR to (2) a sum of the values of each child sub bid that is satisfied and the price associated with the sub bid comprised of the logical operator OR or XOR, wherein said price is included in the sum when said sub bid is satisfied, otherwise it is not included in the sum;

for each sub bid comprised of a logical operator XOR logically connecting the at least two child sub bids, defining:

a seventh mathematical relationship that relates (1) an integer value to (2) a sum of Boolean values related to each child sub bid, wherein each child sub bid that contributes value to the sub bid comprised of the logical operator XOR is assigned a first Boolean value, otherwise it is assigned a second Boolean value, and

an eighth mathematical relationship for each child sub bid that contributes value to the sub bid comprised of the logical operator XOR,

wherein said relationship relates (1) a value of the child sub bid to (2) a product of the Boolean value of said child sub bid times a predetermined value;

and

for each sub bid for k number of child sub bids, where k is less than a total number of child sub bids available, defining:

a ninth mathematical relationship that relates (1) a total number of satisfied child sub bids to (2) a sum of Boolean values related to satisfaction of each child sub bid;

a tenth mathematical relationship that relates (1) a total number of satisfied child sub bids to (2) a product of k times a Boolean value related to satisfaction of the sub bid; and

an eleventh mathematical relationship that relates (1) a value of the sub bid to (2) a sum of the values of each child sub bid that is satisfied and a price associated with the sub bid, wherein said price is included in the sum when said sub bid is satisfied, otherwise it is not included in the sum.

17.   The method of claim 1, wherein:

for each sub bid comprised of one good and an associated price, said sub bid is satisfied when the one good is allocated to the bid including the sub bid;

for each sub bid comprised of a logical operator AND logically connecting at least two child sub bids, said sub bid is satisfied when all of the child sub bids are satisfied;

for each sub bid comprised of a logical operator OR or XOR logically connecting at least two child sub bids, said sub bid is satisfied when at least one of the child sub bids is satisfied; and

for each sub bid for k number of child sub bids, said sub bid is satisfied when k number of child sub bids are satisfied.

18.   A method of processing a plurality of bids whereupon optimizing software can determine an optimal allocation of goods in a combinatorial auction, wherein each bid

includes a plurality of sub bids and each sub bid is comprised of (1) one good and an associated price or (2) a logical operator connecting at least two child sub bids and a price associated with said logical operator, the method comprising:

(a)   defining for each sub bid comprised of one good and an associated price, the constraints:

$$s \leq x \quad \text{and} \quad v \leq s * p \, ;$$

(b)   defining for each sub bid comprised of a logical operator AND logically connecting at least d child sub bids, the constraints:

$$d * s \leq \sum_{i \leq d} s_i \quad \text{and} \quad v \leq (p * s) + \sum_{i \leq d} v_i \, ;$$

(c)   defining for each sub bid comprised of a logical operator OR or a logical operator XOR logically connecting at least d child sub bids, the constraints:

$$s \leq \sum_{i \leq d} s_i \quad \text{and} \quad v \leq (p * s) + \sum_{i \leq d} v_i \, ;$$

(d)   defining for each sub bid comprised of the logical operator XOR logically connecting the at least d child sub bids, the additional constraints:

$$\sum_{i \leq d} t_i \leq 1 \quad \text{and} \quad v_i \leq maxval * t_i, \text{ for every } i \leq d \, ;$$

(e)   defining for each sub bid for k number of child sub bids, where k is less than a total number of child sub bids available, the constraints:

$$n \leq \sum_{i \leq d} s_i \, , \quad s * k \leq n \quad \text{and} \quad v \leq (p * s) + \sum_{i \leq d} v_i \, ;$$

and

(f)   processing the combinatorial bids subject to the constraints defined in steps (a) – (e) to achieve a predetermined objective, wherein:

s = a Boolean variable related to satisfaction of the sub bid;

x = a Boolean variable related to whether the one good has been allocated to the bid including the sub bid;

v = an integer or real variable related to the value of the sub bid;

p = a price associated with the sub bid;

d = an integer value related to the number of child sub bids logically connected by the corresponding logical operator;

i = an integer value related to a particular child sub bid;

s_i = a Boolean variable related to satisfaction of child sub bid i;

v_i = a variable related to the value of child sub bid i;

t_i = a Boolean variable utilized to ensure that the value of only one of the XOR'ed child sub bids contributes to the value (v) for the sub bid;

maxval = a constant having a value greater than any value v_i;

n = an integer or real value related to the number of satisfied child sub bids; and

k = the number of sub bids which, when satisfied, will satisfy the bidder's requirement.

19.   The method of claim 18, wherein:

for each sub bid comprised of one good and an associated price, said sub bid is satisfied when the one good is allocated to the bid including the sub bid;

for each sub bid comprised of a logical operator AND logically connecting at least two child sub bids, said sub bid is satisfied when all of the child sub bids are satisfied;

for each sub bid comprised of a logical operator OR or XOR logically connecting at least two child sub bids, said sub bid is satisfied when at least one of the child sub bids is satisfied; and

for each sub bid for k number of child sub bids, said sub bid is satisfied when k number of child sub bids are satisfied.

20.   The method of claim 18, wherein the predetermined objective is one of maximize and minimize a value of the plurality of bids.

21.   The method of claim 18, wherein in step (f), the plurality of bids is processed utilizing one of (1) an integer program (IP) optimizing software and (2) a mixed integer program (MIP) optimizing software.

22.   A computer-readable medium having stored thereon instruction which, when executed by a processor, cause the processor to perform the steps of:

(a)   receive a plurality of bids each of which includes a plurality of sub bids, wherein each sub bid is comprised of one of (1) one good and an associated price and (2) a logical operator logically connecting at least two child sub bids and a price associated with the logical operator;

(b)   define an objective for the plurality of bids;

(c)   define for each bid a plurality of mathematical relationships without logical operators, wherein said mathematical relationships collectively represent the bid; and

(d)   process the received bids subject to the mathematical relationships to achieve the objective.

23.   The computer-readable medium of claim 22, wherein step (c) includes:

for each sub bid comprised of one good and an associated price, define:

a first mathematical relationship between a pair of Boolean variables that relate (1) the one good being allocated to the bid that includes the sub bid to (2) satisfaction of the sub bid, wherein the sub bid is satisfied when the one good is allocated thereto, and

a second mathematical relationship that relates (1) a value of the sub bid to (2) a product of the price of the sub bid times a value of a Boolean variable related to the satisfaction of the sub bid;

for each sub bid comprised of a logical operator AND logically connecting at least two child sub bids, define:

a third mathematical relationship that relates (1) a sum of Boolean values related to satisfaction of each child sub bid to (2) a product of the total number of the child sub bids logically connected by the logical operator AND times a Boolean value related to the satisfaction of the sub bid comprised of the logical operator AND, wherein the sub bid comprised of the logical operator AND is satisfied when all of the child sub bids logically connected thereby are satisfied, and

a fourth mathematical relationship that relates (1) a value of the sub bid comprised of the logical operator AND to (2) a sum of the values of

each child sub bid that is satisfied and the price associated with the sub bid comprised of the logical operator AND, wherein said price is included in the sum when said sub bid is satisfied, otherwise it is not included in the sum;

for each sub bid comprised of a logical operator OR or XOR logically connecting at least two child sub bids, define:

a fifth mathematical relationship that relates (1) a sum of Boolean values related to satisfaction of each child sub bid to (2) satisfaction of the sub bid comprised of the logical operator OR or XOR, wherein the sub bid comprised of the logical operator OR or XOR is satisfied when at least one of the child sub bids logically connected thereby is satisfied, and

a sixth mathematical relationship that relates (1) a value of the sub bid comprised of the logical operator OR or XOR to (2) a sum of the values of each child sub bid that is satisfied and the price associated with the sub bid comprised of the logical operator OR or XOR, wherein said price is included in the sum when said sub bid is satisfied, otherwise it is not included in the sum;

for each sub bid comprised of a logical operator XOR logically connecting the at least two child sub bids, define:

a seventh mathematical relationship that relates (1) an integer value to (2) a sum of Boolean values related to each child sub bid, wherein each child sub bid that contributes value to the sub bid comprised of the logical operator XOR is assigned a first Boolean value, otherwise it is assigned a second Boolean value, and

an eighth mathematical relationship for each child sub bid that contributes value to the sub bid comprised of the logical operator XOR, wherein said relationship relates (1) a value of the child sub bid to (2) a product of the Boolean value of said child sub bid times a predetermined value;

and

for each sub bid for k number of child sub bids, where k is less than a total number of child sub bids available, define:

a ninth mathematical relationship that relates (1) a total number of satisfied child sub bids to (2) a sum of Boolean values related to satisfaction of each child sub bid;

a tenth mathematical relationship that relates (1) a total number of satisfied child sub bids to (2) a product of k times a Boolean value related to satisfaction of the sub bid; and

an eleventh mathematical relationship that relates (1) a value of the sub bid to (2) a sum of the values of each child sub bid that is satisfied and a price associated with the sub bid, wherein said price is included in the sum when said sub bid is satisfied, otherwise it is not included in the sum.